of the other linear combination of these two states which is orthogonal to the one produced. In the present model, this decoupled state is K_W defined by Eq. (2c). It is hard to believe that this state should have the same mass as the K^0 and $\bar{K}^0$, despite the difference in strong and electromagnetic interactions such as those responsible for the K^0–$K^{\pm}$ mass difference or the πK mass difference. It is much easier to believe that there is a CP nonconservation.

[1]D. I. Lalovic, Phys. Rev. Letters 24, 1662 (1968).

[2]A. Abashian and H. J. Lipkin, Phys. Letters 14, 151 (1965).

[3]J. Uretsky, Phys. Letters 14, 154 (1965).

[4]J. Prentki, in Proceedings of the Oxford International Conference on Elementary Particles, September, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966), p. 48, points out that interference experiments rule out all theories with extra hadrons. The possibility of an extra kaon which only interacts weakly is presumably so far-fetched that it is not even discussed. Since the Oxford Conference, there has been no further mention of extra kaon models in review at international conferences.

EXCHANGE DEGENERACY AND SU(3) FOR BARYONS*

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It is assumed that the contributions of baryon Regge trajectories cancel in the imaginary parts of meson-baryon scattering amplitudes in states of the SU(3) representations $\underline{10}^*$ and $\underline{27}$. Predictions concerning interaction ratios, mass splitting, and particle mixing are made and compared with experiment.

The following "exchange-degeneracy hypothesis" has been useful recently in particle physics: In the imaginary part of any two-hadron scattering amplitude of internal quantum numbers for which no resonances exist, the contributions of t-channel Regge trajectories cancel and those of u-channel trajectories also cancel.[1,2] This usually requires "exchange" degeneracy of trajectories corresponding to physical particles of opposite parity and spins differing by unity. In this paper, we apply the hypothesis to the contributions of baryon trajectories to the scattering of pseudoscalar mesons (P) from $j^P=\frac{1}{2}^+$ baryons (B). Approximate SU(3) symmetry is assumed; the SU(3) implications are surprising because of the different multiplets observed for baryons of even and odd parities.

We first consider the limit of exact SU(3) symmetry. Schmid has pointed out that the $j^P=\frac{5}{2}^-$ $\Sigma^*(1765)$ lies very close to the trajectory of the $j^P=\frac{3}{2}^+$ $\Sigma^*(1382)$, and that this is implied by the exchange-degeneracy hypothesis and the absence of resonances in KN states.[3] Extended to SU(3), the argument implies that the contributions of the $j^P=\frac{3}{2}^+$ decuplet trajectory [denoted by $(\frac{3}{2}^+, \underline{10})$] and the $(\frac{5}{2}^-, \underline{8})$ trajectory must cancel in all PB states of the representations $\underline{10}^*$ and $\underline{27}$. We denote by $R_i=\beta_{i,\underline{10}^*}/\beta_{i,\underline{27}}$ the residue ratio in PB states of representations $\underline{10}^*$ and $\underline{27}$ of a trajectory of SU(3) multiplet i. Our hypothesis implies that the R values of the two exchange-degenerate multiplets must be the same. It can be seen from the octet-octet crossing matrix that $R_{10}=3$ while R_8 is given in terms of the F/D ratio by the formula

$$R_8=(2-6F)/(1+3F^2), \tag{1}$$

where we take $D=1$ so $F=F/D$.[4] The value $R=3$ corresponds to a double root of F, with the value $F=-\frac{1}{3}$. We assume throughout the paper that the ratios of particle-trajectory couplings of the same spin structure are independent of momentum transfer and so are the same in the exchange region and the physical region of the resonance decays. The value $F=-\frac{1}{3}$ is not far from the value -0.14 that is determined from the decays of the $(\frac{5}{2}^-, \underline{8})$ particles.[5,6]

A model that fits very well the observed quantum numbers of resonances is the quark model, or the closely related $SU(6)_W$ model.[7] In quark terminology, the quantum numbers of the lth level of resonances are obtained by adding l units of orbital angular momentum to the basic SU(6) representation $\underline{56}$ (if l is even) or $\underline{70}$ (if l is odd). The spin-SU(3) structure of the $\underline{70}$, and the F values that follow from $SU(6)_W$ symmetry, are[7]

$$(\tfrac{1}{2}, \underline{1}),\quad (\tfrac{1}{2}, \underline{10}),\quad (\tfrac{1}{2}, \underline{8})_{F=5/3},\quad (\tfrac{3}{2}, \underline{8})_{F=-\frac{1}{3}}. \tag{2}$$

Since the physical multiplet $(\frac{5}{2}^-, \underline{8})$ corresponds to the $(\frac{3}{2}, \underline{8})$ of the $\underline{70}$, the exchange-degeneracy prediction agrees with the quark model.

In the quark scheme, the candidates for exchange-degenerate partners with the basic $(\frac{1}{2}^+, \underline{8})$ baryon multiplet are $(\frac{3}{2}^-, \underline{8})$ twice, $(\frac{3}{2}^-, \underline{1})$, and $(\frac{3}{2}^-, \underline{10})$. It is seen from Eq. (1) that the ratio R_8 that corresponds to the SU(6) value of $F = \frac{2}{3}$ for the basic baryon octet is $R_8 = -6/7$, and that this value corresponds to one other value of F, $F = 5/3$. This is the other F of the SU(6)$_W$ model, as seen from Eq. (2). Experimentally, there is solid evidence for one $j^P = \frac{3}{2}^-$ octet, and a phase measurement indicates that $F > 1$ for this octet.[8,6]

The $\frac{3}{2}^-$ singlet and a state of the decuplet also have been discovered.[9,10] It follows from the octet-octet crossing matrix that the R value for a singlet-decuplet mixture is

$$R_{1+10} = (-C_1^2 + 2C_{10}^2)/(C_1^2 + \tfrac{2}{3}C_{10}^2), \tag{3}$$

where C_i^2 is the sum of the squares of the couplings of a state of multiplet i with all PB states.[4] The condition $R = -6/7$ implies $C_{10}^2/C_1^2 = 1/18$.

Our results suggest that there are two types of exchange-degenerate Regge trajectories, represented schematically for an angular-momentum interval of two by the diagram

$$\underline{10}^+ \xrightarrow{\Delta j=1} \underline{8}^-(-\tfrac{1}{3}) \xrightarrow{\Delta j=1} \underline{10}^+, \quad \underline{8}^+(\tfrac{2}{3}) \xrightarrow{\Delta j=1} \left\{ \begin{matrix} \underline{8}^-(5/3) \\ \underline{1}^- + (1/18)\underline{10}^- \end{matrix} \right\} \xrightarrow{\Delta j=1} \underline{8}^+(\tfrac{2}{3}). \tag{4}$$

The sign is the parity, and the number in parenthesis for an octet is the F value. On the second trajectory, the sum of the residues of the odd-parity octet and singlet-decuplet combination should compensate for the even-parity octet in $\underline{10}^*$ and $\underline{27}$ scattering states. Consistency could be obtained if F for the $\underline{8}^-$ on the lower trajectory and $(C_{10}/C_1)^2$ were different from the above values, provided that the ratio $\beta_{\underline{10}^*}/\beta_{\underline{27}}$ of the sum of the octet, singlet, and decuplet were unchanged There is not much room for this type of variation though, since both the Σ^*KN interactions of these odd-parity multiplets must be small.

In the quark model, one of the predicted $l = 2$ multiplets is of type $(\frac{5}{2}^+, \underline{10})$, but no spin-$\frac{1}{2}$ realizations of the corresponding Regge trajectories exist. According to our scheme, the $j = \frac{3}{2}$ multiplet on the exchange-degenerate trajectory is $(\frac{3}{2}^-, \underline{8}_{F=-\frac{1}{3}})$. This multiplet is predicted in the quark model, but its physical existence is uncertain.[11]

We now calculate some partial widths for the $j^P = \frac{3}{2}^-$ resonances, assuming a simple k^5/M^4 phase space factor for the D-wave PB decays, where M is the resonance mass. We assume singlet-octet mixing for the $\Lambda(1519)$ and $\Lambda(1695)$, with a mixing angle θ whose sign is chosen to best fit the data and whose magnitude is determined from the assumption that the octet state and the $N^*(1525)$, $\Sigma^*(1660)$, and $\Xi^*(1815)$ satisfy the Gell-Mann–Okubo linear mass formula.[12] This leads to $\theta = 21°$, with the $\Lambda(1519)$ primarily a singlet. We take F for the octet to be 5/3 (as predicted earlier), and the amplitude C_8 of the octet coupling from the measured $N^*(1525) \to \pi N$ decay.[13] If C_1 is chosen so that the $\Lambda(1519) \to \pi\Sigma$ decay is fitted, then $C_8/C_1 = 0.50$, and the other $\pi\Sigma$ and $\overline{K}N$ widths of the two Λ's may be calculated. The results are compared with experiment in Table I.

We may use the value of C_1 determined above to test the prediction $(C_{10}/C_1)^2 = 1/18$ by assuming that the D_{33} πN resonance observed at 1691 MeV by Donnachie, Kirsopp, and Lovelace, is a member of our $(\frac{3}{2}^-, \underline{10})$ multiplet.[10] The predicted and experimental πN partial widths are 27 MeV and ~37 MeV, respectively.

One may take into account symmetry breaking in applying the exchange-degeneracy hypothesis by considering separately PB states with I_z and Y quantum numbers that do not correspond to the representation $\underline{10}$. This procedure leads to the prediction that the Σ^*'s of the $(\frac{3}{2}^-, \underline{8})$ and $(\frac{3}{2}^-, \underline{10})$ should be nearly degenerate, and the two Ξ^*'s should be nearly degenerate. The experimental situation with respect to these particles is not yet clear. The branching ratios will depend on the mixing, but in the case of degeneracy, the ratios of sums of the decays rates of the two Σ's and of the two Ξ's (uncorrected for phase space

Table I. Experimental and predicted partial widths of Λ^* particles, in MeV.

	$\Lambda^*(1519)$		$\Lambda^*(1695)$	
	$\overline{K}N$	$\pi\Sigma$	$\overline{K}N$	$\pi\Sigma$
Experimental	7.2	7.2	8	23
Predicted	6.2	7.2 (input)	7.7	22.5

differences) are

$$\Sigma^*(\pi\Sigma:\pi\Lambda:\bar{K}N) = 1:0.15:0.10,$$

$$\Xi^*(\bar{K}\Sigma:\bar{K}\Lambda:\pi\Xi) = 1:0.32:0.15,$$

where the coupling ratio $(C_8/C_{10})^2$ has been taken from $(C_{10}/C_1)^2 = 1/18$ and the value of C_8/C_1 in Table I.

We next consider the possible consequences of the mass splitting of the basic Σ and Λ, a splitting that is preserved (and increased somewhat) for the $j=\frac{5}{2}$ recurrences of these particles.[13] We assume exact SU(3) symmetry of the interaction constants. Strict application of the exchange-degeneracy hypothesis leads to the following results: The four Λ and Σ^0 particles of the $j^P=\frac{3}{2}^-$ octet, singlet, and decuplet should be classifiable in two sets, a light set on the $\Lambda(1116)$ trajectory, and a heavy set on the $\Sigma(1193)$ trajectory. Application of the principle to such states as pK^0 (where Σ^+ exchange contributes) implies that the two $\frac{3}{2}^-$ Σ's should lie on the $\Sigma(1193)$ trajectory, so the possibilities for the light set are one or both of the $\frac{3}{2}^-$ Λ's. If the contributions of the light-set particles are to cancel the $\Lambda(1116)$ contribution for the K^+p, $\pi^+\Sigma^+$, and $\bar{K}^0\Xi^0$ states, then the ratio of light-set/heavy-set contributions to each of these states must be the same as the ratio of $\Lambda(1116)/\Sigma^0(1193)$ contributions. The various relevant $\Lambda/(\Lambda+\Sigma^0)$ ratios are given in Table II. Since all the $\Lambda/(\Lambda+\Sigma^0)$ ratios of the table for the K^+p state are 27/28, the entire odd-parity contribution to the K^+p state must result from the light set. One possibility is for both Λ's to be in the light set. In this case, if C_8/C_1 is of such a value that the light/heavy ratio for the $\pi^+\Sigma^+$ state is equal to the required $\frac{3}{4}$, the light/heavy ratio for the $\bar{K}^0\Xi^0$ state is 1.7, which is far from the value 3/25 that corresponds to the even-parity Λ and Σ^0.

The other possibility, one Λ in the light set, satisfies the K^+p requirement if the mixing is such that the heavy Λ is not coupled to the $\bar{K}N$ state. This implies the condition $(C_8/C_1)\cot\theta = (5/12)^{1/2}$. The proper light-set/heavy-set ratio for the couplings to the $\pi^+\Sigma^+$ state will be obtained simultaneously if the further condition $(C_8/C_1)\tan\theta = \frac{1}{3}(5/12)^{1/2}$ is satisfied. In such a case, the light/heavy ratio for the $\bar{K}^0\Xi^0$ state is 0.49, a much better (though not perfect) fit. The two conditions listed above imply the values $C_8/C_1 = 0.37$ and $\theta = 30°$, which are not far from the values of 0.50 and 21° used in Table I.[14] [Actually, the $\Sigma(1660)$ and $\Lambda(1695)$ probably lie above the $\Sigma(1193)$ trajectory, as this trajectory crosses $j=\frac{3}{2}$ at 1590 MeV if it is linear in energy-squared.]

Table II. The $\Lambda/(\Lambda+\Sigma^0)$ coupling ratios, multiplied by 28, for the $j^P=\frac{1}{2}^+$ $(F=\frac{2}{3})$ and $\frac{3}{2}^-$ $(F=5/3)$ octets, and for the $\frac{3}{2}^-$ singlet-decuplet combination.

	K^+p state	$\pi^+\Sigma^+$ state	$\bar{K}^0\Xi^0$ state
$F=\frac{2}{3}$	27	12	3
$F=5/3$	27	3	12
$\underline{1}+(1/18)\underline{10}$	27	27	27

The Λ and Σ^0 also contribute to the following three inelastic amplitudes in channels with no resonances: $K^+n\to K^0p$, $K^-\Sigma^-\to\pi^-\Xi^-$, and $\bar{K}^0\Sigma^+\to\pi^+\Xi^0$. Application of the exchange-degeneracy principle to the first of these reinforces the conclusion that a "heavy-set" Λ should not couple to the $\bar{K}N$ state, while application to the other two processes requires that the relative $K\Xi/\pi\Sigma$ phases of the $\Lambda(1116)$ and $\Lambda^*(1519)$ are the same, a condition that is satisfied by our mixing parameters.[15]

The above argument implies that whenever an even-parity Σ and Λ are split, the odd-parity Λ on the trajectory of the even-parity Λ should be mostly singlet with an octet component of the sign that leads to an increased $\bar{K}N/\pi\Sigma$ interaction ratio. This kind of mixture is observed for the $j^P=\frac{1}{2}^-$ $\Lambda(1405)$, although in this case the even-parity $(j=\frac{3}{2})$ states on the trajectory are not yet discovered.[16]

One of the most important experimental consequences of our predictions is that of the odd-parity Λ's and Σ's, only the Σ's associated with the even-parity decuplet trajectory and the mixed Λ's that are primarily SU(3) singlets should have large branching fractions to $\bar{K}N$ states.

If nature were different in one aspect, i.e., if the baryon resonances of odd "quark-model level" l corresponded to the representation $(\underline{56}, 2l+1)$ of SU(6)⊗O(3), rather than $(\underline{70}, 2l+1)$, the exchange-degenerate baryon partners could belong to the same SU(3) representations, so that the cancellation conditions of this paper could be satisfied more simply. However, in such an alternate world, baryon-trajectory cancellation in the representations $\underline{10}^*$ and $\underline{27}$ would imply cancellation in all other PB representations as well. Thus, combination of the exchange-degeneracy hypothesis with the bootstrap idea that baryon-exchange forces help produce resonances provides a possible dynamical reason for the physically observed alternation of the SU(6) representations 56 and 70.

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[1]This is a generalization of the original exchange-degeneracy hypothesis of R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

[2]See, for example, C. B. Chiu and J. Finkelstein, Phys. Letters 27B, 510 (1968). Physical arguments for the hypotehsis are discussed in this reference.

[3]C. Schmid, to be published.

[4]The octet-octet crossing matrix is given by R. E. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963). Our F parameter is related to the angle θ of this reference by $F=(5/9)^{1/2}\tan\theta$.

[5]See J. Alitti et al., Phys. Rev. Letters 21, 1119 (1968).

[6]The parameter α of Refs. 5 and 8 is related to F by $F\alpha/(1-\alpha)$.

[7]The predictions of an $SU(6)_W$-symmetric potential model for the lighter odd-parity baryon resonances are given by R. H. Capps, Phys. Rev. 158, 1433 (1967).

[8]Anne Kernan and Wesley B. Smart, Phys. Rev. Letters 17, 832 (1966).

[9]For a discussion of the $j^P=\frac{3}{2}^-$ singlet, see R. D. Tripp, R. O. Bangerter, A. Barbaro-Galtieri, and T. S. Mast, Phys. Rev. Letters 21, 1721 (1968).

[10]Evidence for the Δ^* state of the $(\frac{3}{2}^-, \underline{10})$ is given by A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters 26B, 161 (1968).

[11]In the recent review of Haim Harari [in Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September, 1968, (CERN Scientific Information Service, Geneva, Switzerland, 1968)] it is shown that solid experimental evidence exists for at least one resonance of quantum numbers appropriate for membership in every $l=1$ quark-model multiplet except the second $j^P=\frac{3}{2}^-$ octet. See Figs. 5-8 and 11 of this reference.

[12]The strength of the experimental evidence that some such mixing occurs is discussed by G. B. Yodh, Phys. Rev. Letters 18, 810 (1967).

[13]Except where otherwise noted, the masses and partial widths of resonances are taken from the recent compilation of N. Barash-Schmidt et. al., Lawrence Radiation Laboratory Report No. UCRL-8030 Revised, 1968 (unpublished).

[14]Using a similar method of analysis for PP scattering, Chiu and Finkelstein (Ref. 2) argue that the mixing angles for both the vector and tensor meson nonets should be $\theta=\tan^{-1}(2)^{-1/2}$.

[15]The relative $\bar{K}N/\pi\Sigma$ phases of the $\Lambda(1116)$ and $\Lambda^*(1519)$ in our scheme are opposite, but this does not violate the exchange-degeneracy hypothesis. The predicted $\Lambda(1519)$ phase agrees with the measurement of Tripp et al., Ref. 9.

[16]C. Weil, Phys. Rev. 161, 1682 (1967), and J. K. Kim and F. von Hippel (to be published). See Ref. 9 for a further discussion of the $\Lambda^*(1405)$.